Some Dynamical Aspects of a Test Theory of Special Relativity

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Kinematically viable space-time theories admitting a velocity-dependent dilatation factor in addition to the Lorentz transformation between the inertial systems are considered. It is shown that these theories are very unsatisfactory, in the sense of leading neither to a unique definition of time nor to a unique formulation of a dynamics. As an example, the relativistic theory of anisotropic space-time proposed by Bogoslovsky is shown to differ discretely from the special theory of relativity. First-order rotor experiments restrict the free parameter r in this theory to values smaller than 10^{-10}.

Introduction

The special theory of relativity has been accepted universally in physics and it is believed to be the correct theory of space-time transformations between inertial frames of reference. There is, however, a need for a framework in which one can express this belief in specific numbers. In an attempt to give such a framework, theories deviating from the special theory of relativity have been parametrized and a thorough discussion of relevant experiments has been given [1]. It has been shown that the general form of space-time transformations between two inertial systems can be written as [1 a]

$$t = a(v)\,T + \varepsilon(v)\,\varkappa\,, \qquad \varkappa = b(v)(X - v\,T)\,. \tag{1}$$

In the course of making the discussion as simple as possible, we confine ourselves to just one space dimension. Here (T, X) are coordinates in a preferred reference frame Σ in which the velocity of light is isotropic and synchronization by the Einstein Method S_{E} and by slow clock transport S_{T} agree with each other. (t, x) are coordinates in an inertial frame S having a velocity v along the X-axis relative to Σ. The parameters $a(v)$ and $b(v)$ are time dilatation and length contraction factors, respectively. $\varepsilon(v)$ is a pure conventional parameter and is defined by the choice of a synchronization procedure of distant clocks in S. Synchronizing the clocks in an S-frame according to Einstein (ε_{E}) or by the method of slow clock transport (ε_{T}) we get [1 a]

$$\varepsilon_{\mathrm{E}} = -\frac{a}{b(1 - v^2)}\,, \tag{2}$$

$$\varepsilon_{\mathrm{T}} = \frac{1}{b}\,\frac{\mathrm{d}}{\mathrm{d}v}\,a\,. \tag{3}$$

Another synchronization procedure which has been discussed very often in the literature [2, 3] is the so-called "shaft-synchronization". Consider a long, straight shaft of circular cross section mounted on frictionless bearings at rest in an inertial frame of reference. A narrow, straight yellow line is painted on the shaft. Counting devices A and B are placed at two different positions along the shaft (on a straight line parallel to the axis of the shaft). Each device records the passage of the yellow line and displays the number of passages on a dial. The shaft is initially at rest with A and B dials set at zero. A torque is applied for a short time, setting the shaft into rotation. Eventually all torsional vibrations damp out and the shaft is rotating uniformly with a constant angular velocity. Now the numbers on the dials at A and B can be translated into clock readings. Obviously, incorporating this procedure into our test theory means to make severe dynamical assumptions concerning the behaviour of a matter circular shaft under rotation, which is far beyond the scope of a test theory.

To see some of the difficulties in connection with formulating a dynamics in a space-time theory deviating from the special theory of relativity, we make further simplifying assumptions. The velocity of light should be isotropic and independent of the velocity of the frame of reference. Furthermore, we choose S_{E} to synchronize the clocks in S. These assumptions lead to [1, 4]

$$a = d(v)\cdot\sqrt{1 - v^2}\,, \quad b = d(v)/\sqrt{1 - v^2}$$

or

$$t = d(v)\cdot\frac{T - v\,X}{\sqrt{1 - v^2}}\,, \quad \varkappa = d(v)\cdot\frac{X - v\,T}{\sqrt{1 - v^2}}\,. \tag{4}$$

The function $d(v)$ is a dilatation factor which appears in addition to the Lorentz part of the

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transformation. In general, due to the deviation of this factor from unity, the principle of relativity is violated. We see this by noting that S_{T} in Σ and in S leads to different results [1 b]. While in Σ, S_{T} and S_{E} are equivalent, the transport synchronization in S leads to a result other than the Einstein procedure. There are, however, cases in which d depends not only on v but also on the direction of the velocity so that $d(v) \neq d(-v)$. In such a case the principle of relativity can still be valid, as it is shown by the example of Section III.

Now first-order tests and time dilatation experiments can be used to restrict the function $d(v)$ [1 b]. The result of this paper can be recast in the assertion that

$$|\delta| < 10^{-7}, \tag{5}$$

where δ is defined by the first coefficient of expansion in

$$d(v) = 1 + \delta v^2 + \cdots. \tag{6}$$

Therefore at the kinematical level of the test theory a suitable chosen dilatation factor cannot be excluded. Obviously, this kinematical possibility makes theories admitting a specific dilatation in addition to the Lorentz transformation tempting to authors looking for theories deviating from the special theory of relativity.

In testing the theory of special relativity, one has to bear in mind that this is not just an "isolated" theory for itself, but it has rather a "constitutional" character for all physical theories. That is to say, every physical theory is required to be a relativistic one. Thanks to the principle of relativity and to the Lorentz transformations, this requirement leads to a unique procedure for formulating relativistic theories. The simplest example is that of motion of a relativistic free particle. While the Lorentz transformations lead to a unique Lagrangian for a free material particle [5], the existence of an additional dilatation factor brings in an infinite number of possibilities. In Sect. II we will discuss this element of uncertainty. Another source of non-uniqueness lies in the definition of "time". In Sect. III we will discuss this point for the case of a relativistic theory of anisotropic spacetime.

II. Langrangian for a Free Massive Particle

In studying the motion of a material particle, we shall start from the principle of least action [5]. To determine the action integral for a free particle, we note that it must depend on the invariants of our transformations. In the special theory of relativity the only invariant is the interval $\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x^2$. In space-time theories we are considering, the situation is different. There we have for a particle moving in a system S the invariant interval $\mathrm{d}s^2 = \mathrm{d}^2(v)(\mathrm{d}t^2 - \mathrm{d}x^2)$, where v is the velocity of S relative to the preferred frame of reference Σ, and the scalar function $\mathrm{d}(v)$ at our disposal. Furthermore if u is the velocity of the particle relative to Σ, then we have $\mathrm{d}s^2 = \mathrm{d}^2(u)(\mathrm{d}t'^2 - \mathrm{d}x'^2)$, where (t', x') are coordinates of a system in which the particle rests. Therefore, the additional scalar function $\mathrm{d}(u)$ is also to be considered. Now we can write the general form of the action for a freely moving massive particle:

$$S = -\alpha d^n(v)\, d^m(u) \int_a^b \mathrm{d}s,$$

where α, n, and m are yet unspecified real numbers. This action integral can be represented as an integral with respect to the time

$$S = \int_{t_1}^{t_2} \mathscr{L}\, \mathrm{d}t$$

where

$$\mathscr{L} = -\alpha c\, d^{n+1}(v)\, d^m(u) \sqrt{1 - (w^2/c^2)}.$$

Here we have inserted the velocity of light c. w is the velocity of the particle in S. To simplify the further discussion, we set $v = 0$, that is, we study our particle from the preferred frame Σ. Then we have

$$\mathscr{L} = -\alpha c\, d^m(u) \sqrt{1 - (u^2/c^2)}. \tag{7}$$

Now consider the classical limit $c \to \infty$, where we require that the Lagrangian (7) goes over into the classical expression $\mathscr{L} = \frac{1}{2} M u^2$, where M is the mass of the particle. It follows then

$$\alpha(\tfrac{1}{2} - m\,\delta) = \tfrac{1}{2} M,$$

where δ is defined by (6), and we have omitted a constant on the left hand side, as it is of no importance in the Lagrangian. This relation between α, m and M does not fix the parameter m. For any choice of m, where $\frac{1}{2} - m\,\delta > 0$, we get an acceptable value for α. The choice of $m = 0$, which would yield $\alpha = M$, as in the special theory of relativity, is in no way cogent. This is an important element of uncertainty in all these theories. In the words of

Popper [6] the "predictive power" of the theory is softened.

Further we can calculate the momentum $p = \partial L / \partial v$. It follows from (7)

$$p = -\alpha \left[m \frac{\mathrm{d}}{\mathrm{d}u} \cdot d(u) d^{m-1}(u) \sqrt{1-u^2} - \frac{u}{\sqrt{1-u^2}} d^m \right],$$

where we have set again $c = 1$. Taking account of the condition $d(v) = d(v^2)$, this relation can be written as

$$p = \alpha \frac{u}{\sqrt{1-u^2}} \left[d(u) - 2m(1-u^2) \frac{\mathrm{d}}{\mathrm{d}u^2} d(u^2) \right] \cdot d^{m-1}(u).$$

We see that in general p is not proportional to the velocity u.

III. Discussion of some Proposed Alternetve Theories

a) Theories which can Already be ruled Out Kinematically

There are many discussions in the literature [3, 7–9] concerning theories which obviously violate the kinematical condition (5). These theories are therefore to be rejected, as they are strongly in conflict with experiment. For example the theory proposed in [7] is equivalent to

$$d(v) = \sqrt{1-v^2} \cong 1 - \tfrac{1}{2} v^2 \quad \text{or} \quad \delta = -\tfrac{1}{2}$$

in contrast to the restriction of $\delta < 10^{-7}$ by the first-order experiments.

b) Relativistic Theories of Anisotropic Spase-Time

Some of the proposed theories are based on the anisotropic space or space-time [10, 11]. The most elaborated and interesting of them is the theory proposed by Bogoslovsky [11]. He develops a theory of locally anisotropic space-time in the framework of a Finsler geometry. This anisotropy, however, manifests itself only for moving objects, and it is therefore of very special nature (a remarkable fact of the theory is that the preferred direction is a light-like one). The propagation of light, for example, is isotropic in every reference frame. That is, no direction of space is preferred, whatsoever, as far as the propagation of light is concerned. This theory written in one space dimension has a simple representation. In our notation it is equivalent to

$$d(v) = \left(\frac{1-v}{1+v} \right)^r,$$

where r is an unspecified parameter. From the discussion in the paper [1 b], we can easily see that r has to satisfy the following condition

$$r < 10^{-10}.$$

This experimental upper bound makes it clear how small any conceivable anisotropy has to be.

The above function satisfies the relation

$$d(v) = 1/d(-v).$$

Because of this property the inverse transformation of (4) is simply obtained by letting $v \to -v$. This is a manifestation of the nonexistence of a preferred reference frame. At first sight this seems to be a paradoxical situation, as we said earlier that there is always a preferred frame, except for $d(v) \equiv 1$. This paradox is resolved, however, by noting that there can be no reference frame transforming as (4) and (8), in which transport synchronization agrees with Einstein procedure. To see this, we expand the coefficients $a(v)$ and $b(v)$ in the form

$$a = 1 + \alpha_1 v + \alpha v^2,$$
$$b = 1 + \beta_1 v + \beta v^2.$$

For the transformations (4), (8) we have

$$\alpha_1 = -2r, \quad \alpha = -\tfrac{1}{2}, \qquad \beta_1 = -2r, \quad \beta = +\tfrac{1}{2}. \tag{9}$$

Transport synchronization yields

$$\varepsilon_{\mathrm{T}} = \frac{1}{b} \cdot \frac{\mathrm{d}}{\mathrm{d}v} a_1 \cong \alpha + (2\alpha - \alpha_1 \beta_1) v,$$

whereas Einstein procedure leads to

$$\varepsilon_{\mathrm{E}} = -v.$$

Setting $v = 0$, we get

$$\varepsilon_{\mathrm{T}} = \alpha_1, \quad \varepsilon_{\mathrm{E}} = 0. \tag{10}$$

As far as $\alpha_1 \neq 0$ these two procedures lead to different results. $v = 0$ means synchronization in the preferred reference frame, or in the case where all systems are equivalent, in any system without recourse to a preferred frame. It is therefore possible to measure the one-way velocity of light, if the clocks are synchronized according to transport procedure. Thus this theory is an example of a relativistic theory of space-time in which the one-way velocity of light is predicted to be different from the two-way velocity c, which enters into the transformation equations as a constant. This fact elucidates a "discrete" difference between this theory and the special theory of relativity: the effect of anisotropic one-way light velocity does not depend on any velocity whatsoever. This theory has to be looked at as another generalization of the Galilei transformation, which differs from the special theory of relativity discretely. Both theories have the common feature of reducing to Galilei transformation for small velocities. Therefore it is misleading to say that this theory differs significantly from special relativity only when the relative velocity of the inertial systems is very close to the velocity of light [11].

There is still another problem which arises in this context. The coordinate "t" as it stands in the transformation equations is shown by clocks at fixed points in space and synchronized according to S_E. Now, we could as well synchronize the clocks according to S_T, which is physically as good as S_E. Then the new coordinate time τ will be shown by slowly moving clocks. But according to (1), (9) and (10) we have the following relation between t and τ:

$$\tau = t - 2 r \varkappa .$$

One sees that these two "time coordinates" agree only at one point. Any dynamics can be formulated in either of these two physically unpreferable "times". Now the question is to which of these alternatives do the actual physical phenomena correspond. There is no a-priori reason to prefer one of these "times". Thus, with respect to dynamical effects, there seems to be no built-in unique predictability in this theory, as opposed to the special theory of relativity, where we have a uniquely defined time coordinate.

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